Discover the Powerful Applications of Forward Backward Stochastic Differential Equations!

Understanding Forward Backward Stochastic Differential Equations

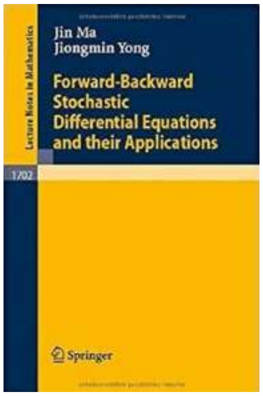
Forward Backward Stochastic Differential Equations (FBSDEs) form a powerful mathematical framework that has found applications in various fields such as finance, engineering, and mathematical physics. FBSDEs combine concepts from stochastic calculus, partial differential equations, and control theory to model complex dynamical systems with both forward and backward components.

Solving Forward Backward Stochastic Differential Equations

Solving FBSDEs involves finding a solution to a coupled system of forward and backward stochastic differential equations. The forward component describes the evolution of a system over time, while the backward component represents the backward time evolution of the system based on a terminal condition. The interplay between these equations allows for the modeling of systems with dependencies on both past and future events.

To solve FBSDEs, one typically employs numerical methods such as finite difference schemes or Monte Carlo simulations. These techniques approximate the solutions by discretizing the time and state variables, enabling the evaluation of the equations at different points in the system's evolution. Advanced computational algorithms are also employed to handle the high-dimensional nature of FBSDE systems, ensuring accurate and efficient solutions.

Forward-Backward Stochastic Differential Equations and their Applications (Lecture Notes in



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Applications of Forward Backward Stochastic Differential Equations

FBSDEs have gained extensive applications in various fields due to their ability to capture complex dynamics and dependencies in systems. Here are a few notable applications:

1. Mathematical Finance

In finance, FBSDEs are used to model and study complex derivative pricing problems, portfolio optimization, risk management, and option hedging strategies. The backward component allows for incorporating future market information into the pricing and risk assessment process, making FBSDEs particularly useful in dynamic financial environments.

2. Stochastic Control and Optimal Control

FBSDEs play a crucial role in stochastic control theory and optimal control problems. These equations are employed to determine the optimal control strategies under uncertain and dynamic environments. The backward component represents the cost-to-go function, while the forward component models the system dynamics. By solving the coupled equations, optimal control policies can be derived, leading to effective decision-making in various control applications.

3. Mathematical Physics

In mathematical physics, FBSDEs find applications in quantum field theory, particle physics, and statistical mechanics. These equations help model and analyze non-equilibrium dynamics, quantum many-body systems, and complex interacting systems. By considering both the forward and backward components, FBSDEs provide a versatile framework for investigating a wide range of physical phenomena.

In

Forward Backward Stochastic Differential Equations offer a powerful mathematical tool for modeling and analyzing complex systems with dependencies on past and future events. Their applications span across diverse disciplines, including finance, control theory, and mathematical physics. By solving the coupled forward and backward equations, valuable insights can be gained, leading to improved decision-making, risk assessment, and understanding of complex dynamics. Embracing the computational techniques associated with FBSDEs allows for effective utilization of this versatile framework in real-world scenarios.

Consider the Black-Scholes model in Section 2.8. Assume a self-financing portfolio (λ, h) hedges ξ. By (2.8.8) and (2.8.7) we have:

$$
\begin{aligned}
dV_t &= \Big[\lambda_t r e^{rt} + h_t S_t \mu\Big]dt + h_t S_t \sigma dB_t \\
&= \Big[r(V_t - h_s S_t) + h_t S_t \mu\Big]dt + h_t S_t \sigma dB_t. \qquad (4.5.1)
\end{aligned}
$$

Denote

$$
Y_t := V_t, \quad Z_t := \sigma S_t h_t. \qquad (4.5.2)
$$

Then (4.5.1) leads to

$$
dY_t = \Big[r[Y_t - \frac{Z_t}{\sigma S_t}] + \frac{\mu Z_t}{\sigma S_t}\Big]dt + Z_t dB_t, \quad Y_T = \xi, \quad \mathbb{P}\text{-a.s.} \qquad (4.5.3)
$$

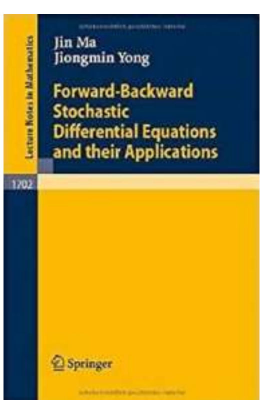
This is a linear BSDE. Once we solve it, we obtain that:

$$
Y \text{ is the price of the option } \xi \text{ and Z induces the hedging portfolio: } h_t = \frac{Z_t}{\sigma S_t}. \qquad (4.5.4)
$$

We remark that BSDE (4.5.3) is under the market measure $\mathbb{P}$. In this approach, there is no need to talk about the risk neutral measure.

Note that BSDE (4.5.3) is linear, which can be solved explicitly. In particular, for the special example we are presenting, Y_0 can be computed via the well-known Black-Scholes formula. To motivate nonlinear BSDEs, let us assume in a more practical manner that the lending interest rate r_1 is less than the borrowing interest rate r_2. That is, the self-financing condition (4.5.1) should be replaced by

$$
dV_t := \Big[r_1(V_t - h_t S_t)^+ - r_2(V_t - h_t S_t)^-\Big]dt + h_t dS_t. \qquad (4.5.5)
$$



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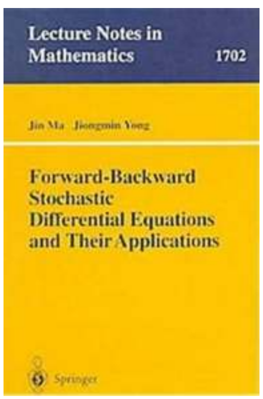
This volume is a survey/monograph on the recently developed theory of forward-backward stochastic differential equations (FBSDEs). Basic techniques such as the method of optimal control, the 'Four Step Scheme', and the method of continuation are presented in full. Related topics such as backward stochastic PDEs and many applications of FBSDEs are also discussed in detail. The volume is suitable for readers with basic knowledge of stochastic differential equations, and some exposure to the stochastic control theory and PDEs. It can be used for researchers and/or senior graduate students in the areas of probability, control theory, mathematical finance, and other related fields.



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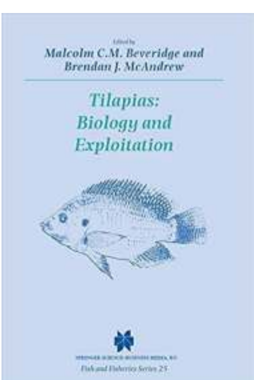

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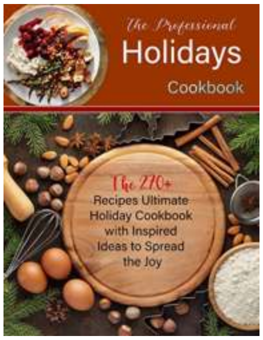

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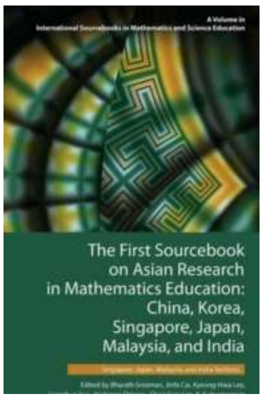

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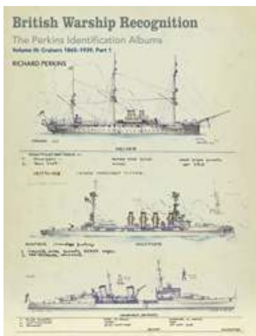

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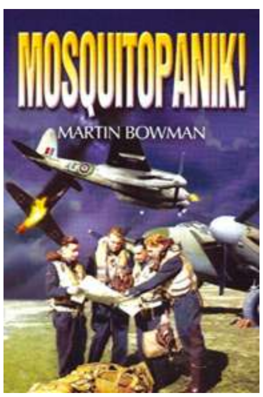

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