The Fascinating Journey of Classical Orthogonal Polynomials: Uncovering the Genius of David Kahn

Classical and Semi-Classical Orthogonal Polynomials Defined by Riordan Arrays, and Their Moment Sequences

Paul Barry and Arnauld Mesinga Mwafise
School of Science
Waterford Institute of Technology
Ireland
pbarry@wit.ie

Abstract

We study the orthogonal polynomials of classical and semi-classical types that can be defined by ordinary and exponential Riordan arrays. We identify their moment sequences, giving their integral representations and Hankel transforms. For a special class of classical orthogonal polynomials defined by Riordan arrays, we identify a complementary family of orthogonal polynomials defined by reversion of moment sequences. Special product sequences arise and their generating functions are calculated.

1 Introduction

Riordan arrays [4, 28, 29, 30, 34] are simple to define (see below), providing a bridge between elements of algebra, group theory and linear algebra. This combination can shed light on other areas of mathematics. In this note, we show how Riordan arrays can yield fresh perspectives on the area of orthogonal polynomials. It is straight-forward to classify those Riordan arrays that define orthogonal polynomials - essentially, they are the ordinary Riordan arrays whose production matrices are tri-diagonal. Nevertheless, it is interesting to note that only in a limited number of cases are the associated orthogonal polynomials of classical type. This note explores this fact. We classify those ordinary Riordan arrays that define classical polynomials, and we study some integral representations of the moment sequences associated with semi-classical orthogonal polynomials defined by ordinary Riordan arrays.

1

Classical orthogonal polynomials have been an integral part of mathematical research for centuries. They have contributed immensely to various scientific

fields, including physics, engineering, computer science, and economics. In this article, we delve deep into the captivating world of classical orthogonal polynomials, highlighting the remarkable accomplishments of the mathematical genius himself, David Kahn.

What are Classical Orthogonal Polynomials?

Classical orthogonal polynomials are a set of polynomials that possess remarkable properties, primarily related to their orthogonality and recurrence relations. These polynomials are typically defined over a certain interval and have specific weight functions associated with them. The most commonly studied classical orthogonal polynomials include Legendre polynomials, Chebyshev polynomials, and Hermite polynomials, among others.

Classical Orthogonal Polynomials, The

by David S. Kahn (Kindle Edition)

★★★★☆ 4.4 out of 5

Language	: English
File size	: 13416 KB
Text-to-Speech	: Enabled
Screen Reader	: Supported
Enhanced typesetting	: Enabled
Print length	: 171 pages
X-Ray for textbooks	: Enabled

FREE **DOWNLOAD E-BOOK** 📄 PDF

The Extraordinary Mind of David Kahn

David Kahn, an eminent mathematician, played a pivotal role in furthering our understanding of classical orthogonal polynomials. His groundbreaking research paved the way for numerous applications of these polynomials in diverse fields.

Kahn's work emphasized the importance of orthogonality, generating an extensive collection of orthogonal polynomials with exceptional properties.

A Glimpse into Kahn's Contributions

One of Kahn's notable contributions to the realm of classical orthogonal polynomials is his discovery of the Bell polynomials. These specialized polynomials, derived from the exponential generating function of the Bell numbers, possess remarkable combinatorial properties. Kahn's Bell polynomials find applications in various areas, including partitions, combinatorial identities, and numerical analysis.

Applications of Classical Orthogonal Polynomials

The widespread utility of classical orthogonal polynomials makes them invaluable in various scientific disciplines. They find applications in solving differential equations, calculus of variations, probability theory, signal processing, numerical integration, interpolation, and approximation theory. The exceptional properties of these polynomials make them particularly effective in accurately representing complex functions.

Further Innovations in the Field

The advancements in classical orthogonal polynomials have continued to evolve over time. Modern researchers have extended the classical polynomials to encompass more diverse scenarios. For instance, the Meixner polynomials and the Laguerre polynomials are extensions of the classical orthogonal polynomials, catering to specific mathematical situations. These innovations have expanded the scope of application for classical orthogonal polynomials.

Inspiration for Aspiring Mathematicians

Kahn's profound contributions to classical orthogonal polynomials serve as an inspiration for budding mathematicians. His dedication to unraveling the mysteries of these polynomials showcases the beauty and elegance of mathematical exploration. Aspiring mathematicians are encouraged to delve into the world of classical orthogonal polynomials, seeking to discover new possibilities and contribute to the ever-growing body of knowledge.

Classical orthogonal polynomials, with their rich history and extensive applications, continue to fascinate and inspire mathematicians worldwide. David Kahn's remarkable contributions have significantly propelled the field forward, shedding light on the remarkable properties of these polynomials. As researchers continue to explore and innovate, classical orthogonal polynomials will undoubtedly remain a cornerstone of mathematical research, enabling breakthroughs in various scientific domains.
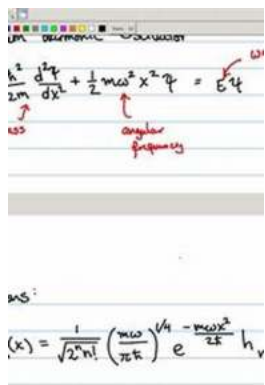
Classical Orthogonal Polynomials, The

by David S. Kahn (Kindle Edition)

★★★★☆ 4.4 out of 5

|	
Language	: English
File size	: 13416 KB
Text-to-Speech	: Enabled
Screen Reader	: Supported
Enhanced typesetting	: Enabled
Print length	: 171 pages
X-Ray for textbooks	: Enabled

This book defines sets of orthogonal polynomials and derives a number of properties satisfied by any such set. It continues by describing the classical orthogonal polynomials and the additional properties they have.The first chapter

defines the orthogonality condition for two functions. It then gives an iterative process to produce a set of polynomials which are orthogonal to one another and then describes a number of properties satisfied by any set of orthogonal polynomials. The classical orthogonal polynomials arise when the weight function in the orthogonality condition has a particular form. These polynomials have a further set of properties and in particular satisfy a second order differential equation. Each subsequent chapter investigates the properties of a particular polynomial set starting from its differential equation.

The Fascinating Journey of Classical Orthogonal Polynomials: Uncovering the Genius of David Kahn

Classical orthogonal polynomials have been an integral part of mathematical research for centuries. They have contributed immensely to various scientific...
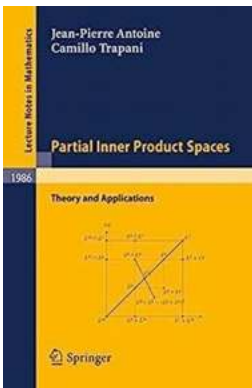
Theory And Applications Lecture Notes In Mathematics 1986: Unveiling the Secrets of Mathematical Marvels

Theory And Applications Lecture Notes In Mathematics 1986: Unveiling the Secrets of Mathematical Marvels Mathematics has long been regarded as the language of the...
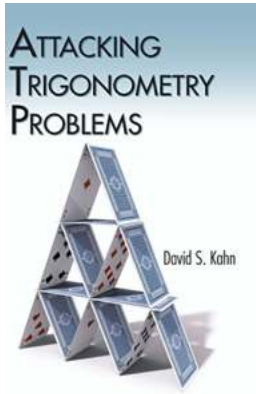
5 Powerful Techniques for Attacking Trigonometry Problems from Dover on Mathematics

to Trigonometry Trigonometry is often regarded as a challenging subject for many students. The complex calculations involved and the extensive use of formulas can be...

The Complete Guide On How To Hunt Chipmunks Including Tools Equipment Supplies

Chipmunks may be small and cute, but for those seeking outdoor adventure, hunting these elusive creatures can provide an exciting experience. However, it's...

Cranberry Lingonberry Huckleberry Berries From Ericaceae Family - The Powerhouse of Nutrients You Need!

When it comes to berries, the Ericaceae family has some of the most delicious and nutritious fruits that you can find. From the tart Cranberry to the tangy...

Unveiling the Secrets to Bringing Home The Ashes and Winning with England

The Battle for the Ashes Every cricket enthusiast understands the significance of the Ashes series. It is the ultimate test of skills and determination between England and...
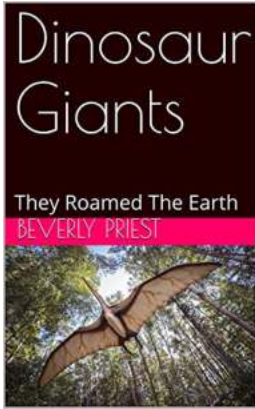
Discover the Astounding Secrets of the Dinosaur Giants That Once Ruled Our Planet!

When we look at the world today with its incredible diversity of life, it's almost hard to imagine that giants once roamed the Earth. Millions of years ago,...
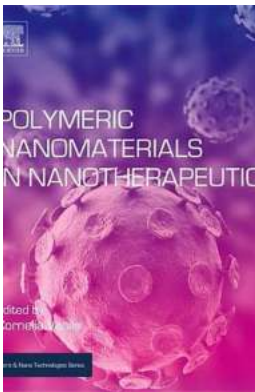
Unveiling the Astonishing Potential of Polymeric Nanomaterials in Nanotherapeutics Micro and Nano Technologies

Polymeric nanomaterials have emerged as a revolutionary technology in the field of nanotherapeutics. With their unique properties and versatile applications,...

classical orthogonal polynomials of a discrete variable

classical orthogonal polynomials a functional approach