

Unveiling the Hidden Powers of Generalized Functions with Applications in Sobolev Spaces!



GENERALIZED DUNKL-SOBOLEV SPACES OF EXPONENTIAL TYPE AND APPLICATIONS

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Received 22 March, 2008; accepted 23 May, 2009

Communicated by S.S. Dragomir

Dedicated to Khalifa Trimèche.

ABSTRACT. We study the Sobolev spaces of exponential type associated with the Dunkl-Bessel Laplace operator. Some properties including completeness and the imbedding theorem are proved. We next introduce a class of symbols of exponential type and the associated pseudo-differential-difference operators, which naturally act on the generalized Dunkl-Sobolev spaces of exponential type. Finally, using the theory of reproducing kernels, some applications are given for these spaces.

Key words and phrases: Dunkl operators, Dunkl-Bessel-Laplace operator, Generalized Dunkl-Sobolev spaces of exponential type, Pseudo differential-difference operators, Reproducing kernels.

2000 Mathematics Subject Classification: Primary 46F15. Secondary 46F12.

1. INTRODUCTION

The Sobolev space $W^{s,p}(\mathbb{R}^d)$ serves as a very useful tool in the theory of partial differential equations, which is defined as follows

$$W^{s,p}(\mathbb{R}^d) = \left\{ u \in \mathcal{S}'(\mathbb{R}^d), (1 + |\xi|^2)^{\frac{s}{2}} \mathcal{F}(u) \in L^p(\mathbb{R}^d) \right\}.$$

In this paper we consider the Dunkl-Bessel Laplace operator $\Delta_{k,\beta}$ defined by

$$\forall x = (x', x_{d+1}) \in \mathbb{R}^d \times]0, +\infty[, \quad \Delta_{k,\beta} = \Delta_{k,x'} + L_{\beta,x_{d+1}}, \quad \beta \geq -\frac{1}{2},$$

where Δ_k is the Dunkl Laplacian on \mathbb{R}^d , and L_β is the Bessel operator on $]0, +\infty[$. We introduce the generalized Dunkl-Sobolev space of exponential type $W_{\mathcal{G},k,\beta}^{s,p}(\mathbb{R}_+^{d+1})$ by replacing

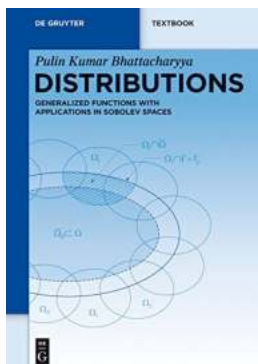
Thanks to the referee for his suggestions and comments.
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The Advent of Generalized Functions

In the vast realm of mathematics, certain phenomena do not conform to traditional functions' behavior, necessitating the development of a new mathematical tool known as generalized functions. These extraordinary mathematical objects find immense utility in various branches of science, engineering, and analysis. One fascinating application lies in the profound relationship between generalized functions and Sobolev spaces.

Understanding Generalized Functions

Generalized functions, also referred to as distributions, are mathematical objects that extend the concept of traditional functions. They provide a framework to analyze situations where conventional functions fail to exist, such as Dirac's delta function. Unlike classical functions, generalized functions do not require pointwise continuity. Instead, they are defined through their interactions with smooth test functions. Through this framework, intricate phenomena, such as impulse responses and distributional derivatives, can be studied.



Distributions: Generalized Functions with Applications in Sobolev Spaces (De Gruyter Textbook)

by Andrew Day ([Print Replica] Kindle Edition)

★★★★★ 5 out of 5

Language : English

File size : 27915 KB

Screen Reader : Supported

Print length : 872 pages



The Power of Sobolev Spaces

Sobolev spaces, named after the eminent mathematician Sergei Sobolev, represent a class of function spaces defined by their derivative regularity. These

spaces provide a platform to study functions with weak and partial derivatives, which are often encountered in problems involving differential equations, optimization, and calculus of variations. Sobolev spaces bring together the aspects of function spaces and distribution theory, enabling the analysis of generalized functions through the lens of functional analysis.

Applications in Science and Engineering

The marriage of generalized functions and Sobolev spaces finds widespread application in various scientific and engineering disciplines. Let's delve into a few notable examples:

1. Signal Processing:

Generalized functions are extensively utilized in signal processing to model and analyze non-ideal signals. The convolution of a generalized function with a signal yields a more accurate representation, allowing noise removal, compression, and better system modeling.

2. Quantum Mechanics:

In quantum mechanics, the wave function is often represented as a generalized function, enabling physicists to study complex phenomena such as wave packets and quantum superposition. This framework helps understand the behavior of particles in various potential energy landscapes.

3. Partial Differential Equations (PDEs):

The use of Sobolev spaces in solving PDEs has revolutionized the field by allowing the treatment of weak solutions. Generalized functions provide an elegant framework to extend the concept of solutions to a broader class of PDEs by considering distributions, facilitating the understanding of complex physical phenomena.

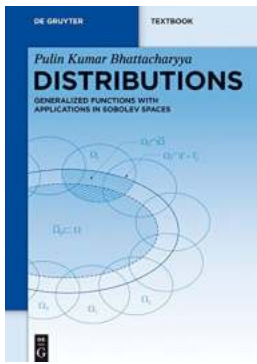
Benefits of Generalized Functions in Sobolev Spaces

The utilization of generalized functions in Sobolev spaces carries several advantages:

- **Enhanced Mathematical Modeling:** The framework of generalized functions enables the modeling of complex and irregular situations, enhancing the accuracy of mathematical models.
- **Wider Class of Test Functions:** Generalized functions allow testing against a broader class of smooth functions, providing more flexibility in analyzing phenomena.
- **Improved Numerical Approximations:** The use of Sobolev spaces allows for more accurate and efficient numerical approximations of problems involving partial derivatives.
- **Better Understanding of Physical Phenomena:** The study of generalized functions in Sobolev spaces aids in comprehending intricate physical phenomena by expanding the notion of solutions.

In Summary

Generalized functions and their applications in Sobolev spaces offer a powerful mathematical framework to study complex and irregular phenomena in various scientific and engineering domains. By extending traditional functions and embracing the concept of weak solutions, they provide more accurate modeling, a broader class of test functions, improved numerical approximations, and a deeper understanding of physical phenomena. The marriage between generalized functions and Sobolev spaces continues to shape the landscape of mathematics, making remarkable contributions to scientific progress.



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This book grew out of a course taught in the Department of Mathematics, Indian Institute of Technology, Delhi, which was tailored to the needs of the applied community of mathematicians, engineers, physicists etc., who were interested in studying the problems of mathematical physics in general and their approximate solutions on computer in particular.

Almost all topics which will be essential for the study of Sobolev spaces and their applications in the elliptic boundary value problems and their finite element approximations are presented. Also many additional topics of interests for specific applied disciplines and engineering, for example, elementary solutions, derivatives of discontinuous functions of several variables, delta-convergent sequences of functions, Fourier series of distributions, convolution system of equations etc. have been included along with many interesting examples.



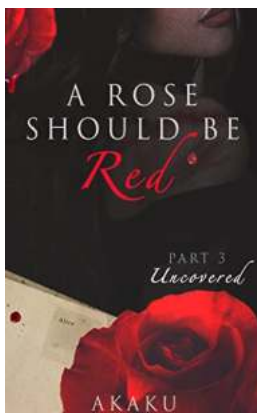
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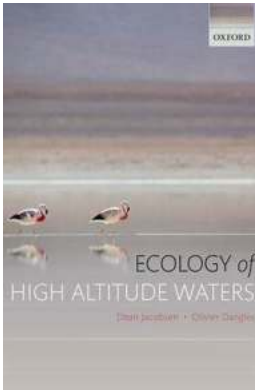
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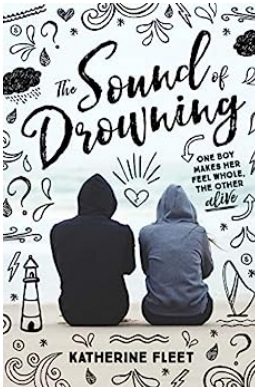
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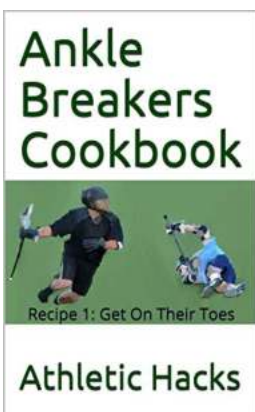
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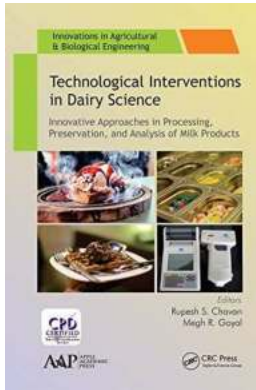
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